How perception and neural weighting can drive the geometry of decision-making in target pursuit

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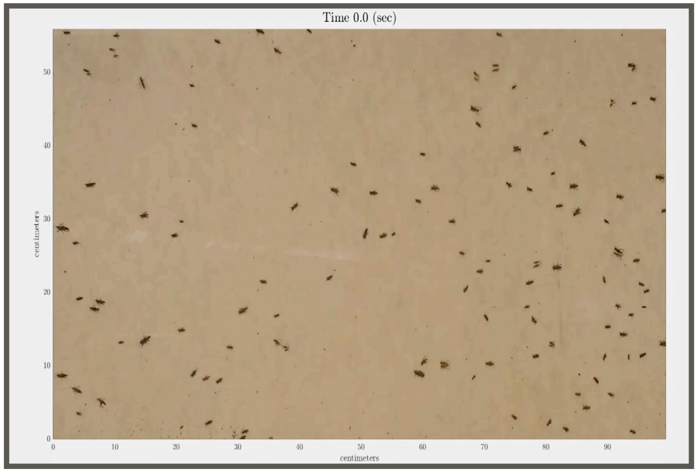
Murmuration of starlings
(2025, my backyard. Video Credit: A. M. Ho)



Weinburd, Landsberg, Kravtsova, Lam, Sharma, Simpson, Sword, Buhl (2024)

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Inspiration for this project: locust collective behavior

Overarching Goal: understand how individual-level mechanisms can result in the behavior and collective action that we see in nature.

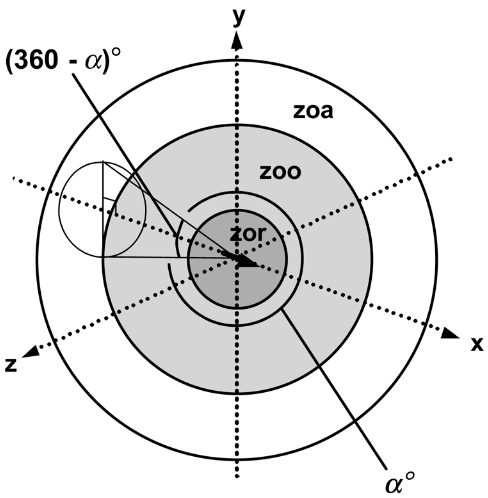
For locusts: we study hopper band formation and behavior

Locusts are significant agricultural pests that exist on every continent. Plagues can be devastating, and control is often challenging – especially in locations with resource scarcity. Understanding hopper bands is key for control efforts – by the time they are flying, it is too late.

Standard behavior models for collective motion

Basic collective motion models include the Vicsek alignment model (1995) and the **three-zone model** of repulsion, alignment, and attraction:

- Each organism is surrounded by 3 zones
- It is repelled by very close organisms
- It wants to *align* with middle-range ones
- It is attracted to far-away organisms



(Couzin *et al.* 2002, *J Theor Biol* 218)

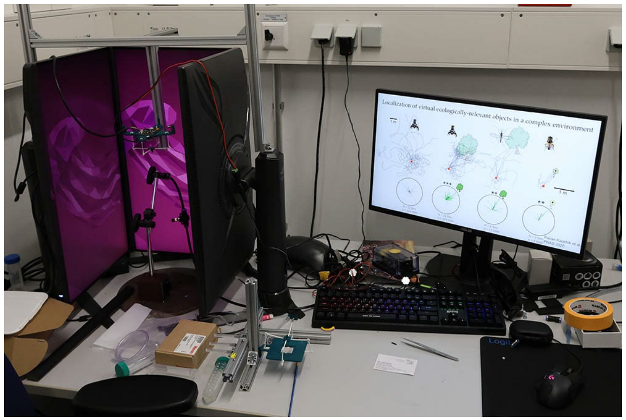
The **alignment** is what is important here – the expectation is that locusts align to move cohesively. Information transferal within groups can also be explained w/ this model (Couzin et al. *Letters to Nature* 2005)

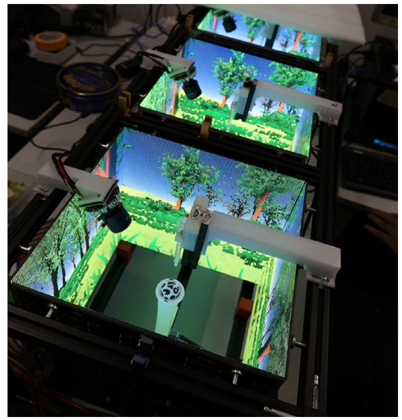
This produces collective behavior if the swarm is *dense enough*. This transition was initially supported by field data (Buhl et al. *Science* 2006)

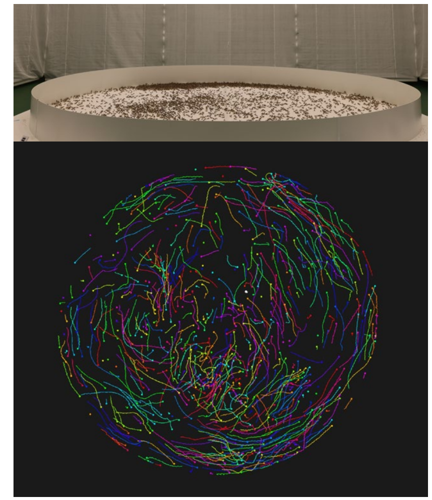
Max Planck Institute for Animal Behavior

Iain Couzin
Max Planck Institute of Animal Behavior
SMB Plenary Speaker!
Friday

For part of my sabbatical, I visited the Dept. of Collective Behavior at the Max Planck Institute of Animal Behavior.







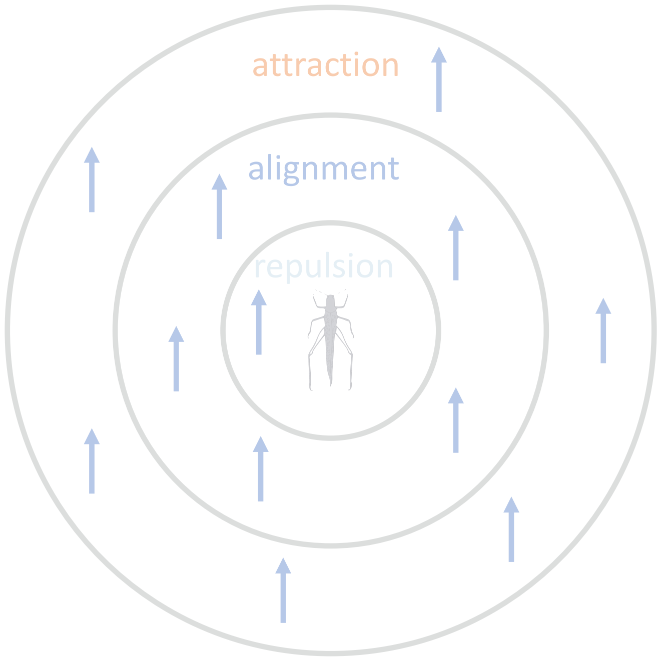
They have done incredible experiments on locusts (and other organisms) suggesting that alignment is not the driving mechanism behind collective motion!

Also: robust movement patterns can be produced from a neural ring model their group was the first to propose.

- Sayin, Couzin-Fuchs, Petelski, Günzel, Salahshour, Lee, Graving, Li, Deussen, Sword, Couzin, (2025) *Science*.
- Gorbonos, Gov, Couzin, (2024) *PRX Life.*
- Sridhar, Li, Gorbonos, Nagy, Schell, Sorochkin, Gov, Couzin, (2021) *PNAS.*

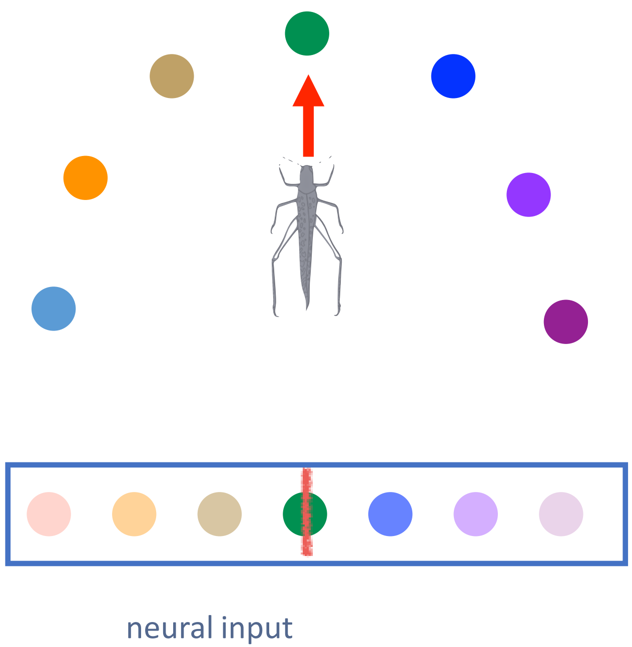
Classical models of collective behavior often take a "bird's-eye perspective," assuming that individuals have access to social information that is not directly available (e.g., the behavior of individuals outside of their field of view).
- Bastien & Romanczuk (2020)

Three Zone Model



pairwise, additive

Neural Band Model

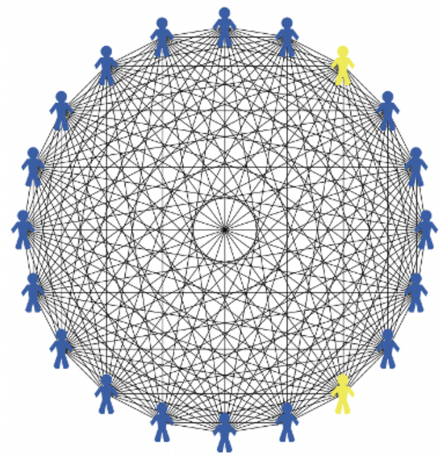


dominant signal wins

Bastien & Romaczuk (2020); Sridhar et al. (2021); Gorbonos, Gov & Couzin (2024); Sayin et al. (2025).

Neural Ring Attractor Network

Imagine a ring of neurons that encode possible locations of an object of interest around you. This is inspired by bat brains, among other species.



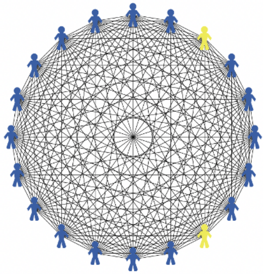
Suppose a locust sees 2 targets (yellow).

⇒ These are the only neuron groups firing.

- These are *angular locations*, no distance to targets is encoded.
- Every neuron is connected to all others, with many neurons for each location.
- Neurons in the target directions fire “on” or “off” randomly due to the stimulus (and interactions with other active neurons).
- Non-target directions are always “off”.

Sridhar et al. (2021); Gorbonos, Gov & Couzin (2024); Sayin et al. (2025).

Neural Ring Attractor Network



Neurons interact via an Ising model with Hamiltonian

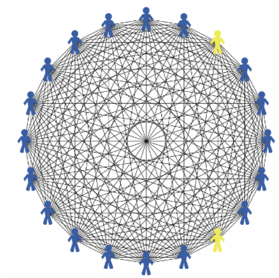
$$\mathcal{H} = -\frac{\mathcal{E}}{N} \sum_{m=1}^{N} \sum_{\substack{n=1, \\ n \neq m}}^{N} \sigma_m \sigma_n J(\theta_m, \theta_n)$$

- N is the total number of stimulated neurons (whether on or off); $\mathcal{E}$ is an energy constant.
- **Each neuron $\boldsymbol{\sigma_m}$** is either on (1) or off (0) with an associated angular group θ_m.
- **J is an *interaction kernel*** between neuron groups: close-by neuron angles excite each other, while far away ones inhibit each other.

A natural example is $J(\theta_m, \theta_n) = \cos(\theta_{m,n})$ where $\theta_{m,n}$ is the shortest angle between θ_m and θ_n.

Sridhar et al. (2021); Gorbonos, Gov & Couzin (2024); Sayin et al. (2025).

Neural Ring Attractor Network



Dynamics are then defined as a Markov process where neurons turn on/off according to Bolzman statistics (Glauber dynamics).

- Rates of a given neuron turning on or off are:

$$r_{0\to 1} = \frac{1}{\tau_0}\frac{1}{1+e^{\Delta\mathcal{H}/k_B T}}\,, \qquad r_{1\to 0} = \frac{1}{\tau_0}\frac{1}{1+e^{-\Delta\mathcal{H}/k_B T}}\,.$$

  τ_0 is the neural noise timescale, k_B the Boltzman constant, T temperature.

- *In the mean-field limit* as $N \to \infty$, one can derive an ODE for the fraction of excited neurons in each angular group.

The mean-field ODE is a **gradient flow**; dynamics are almost always asymptotic to equilibrium states for the neural system.

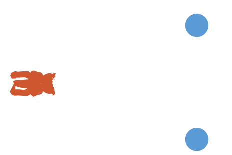
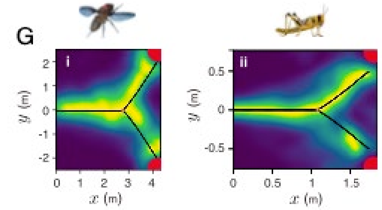
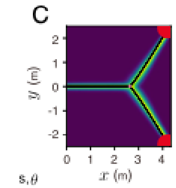
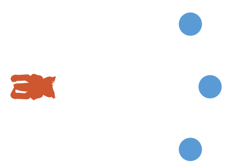
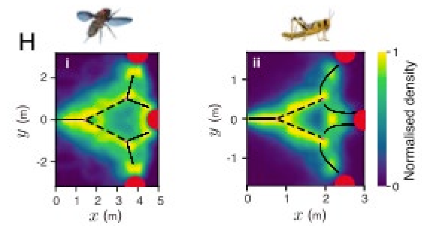
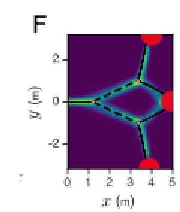
The direction of motion for the organism is therefore taken as the mean direction among all active neurons at neural equilibrium.

Sridhar et al. (2021); Gorbonos, Gov & Couzin (2024); Sayin et al. (2025).

Neural Ring Attractor Network

There may be multiple attracting equilibria. Walkers are assumed to use the previous neural equilibrium as an initial condition for the next one.

The result is a bifurcation pattern in space corresponding to locations where decisions are made. This is seen *across species* in experimental data!

	Geometry	Experiments	Neural Band Model
Two Targets			
Three Targets			

<u>*Note*</u>: Targets are δ-functions, and the cosine kernel is warped to match the bifurcation point with experimental data. *This has mathematical side effects.*

Images from Sridhar, Li, Gorbonos, Nagy, Schell, Sorochkin, Gov, Couzin (2021)

The goals for our project:

1. Extend this modeling framework to targets of positive area/volume (in a simple, natural way).

2. Explore what is possible without warping the cosine kernel.

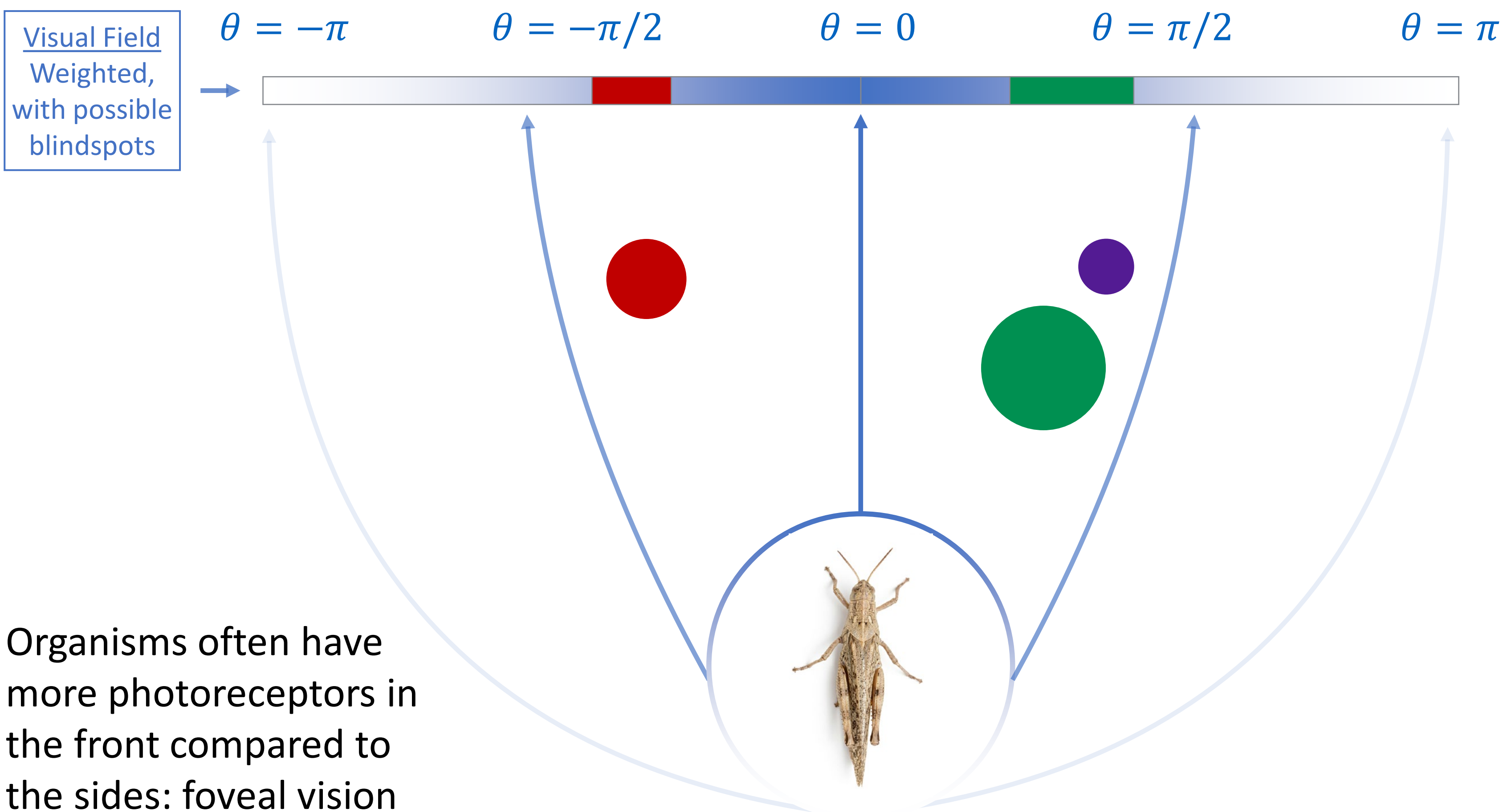
Why? Because the Ising model is *all-to-all* coupling ($\mathcal{O}(N^2)$), but through spectral decomposition, we can show that $J(\theta_m, \theta_n) = \cos(\theta_{m,n})$ is rank 2 (and thus involves $\mathcal{O}(2N)$ couplings) while the warped version is full rank.

To accomplish this, we will introduce the ideas of relative neuron group sizes and a neural density function.

These will have the effect of **weighting** and then **warping** target perception, respectively.

Preprocessing: Target Weighting

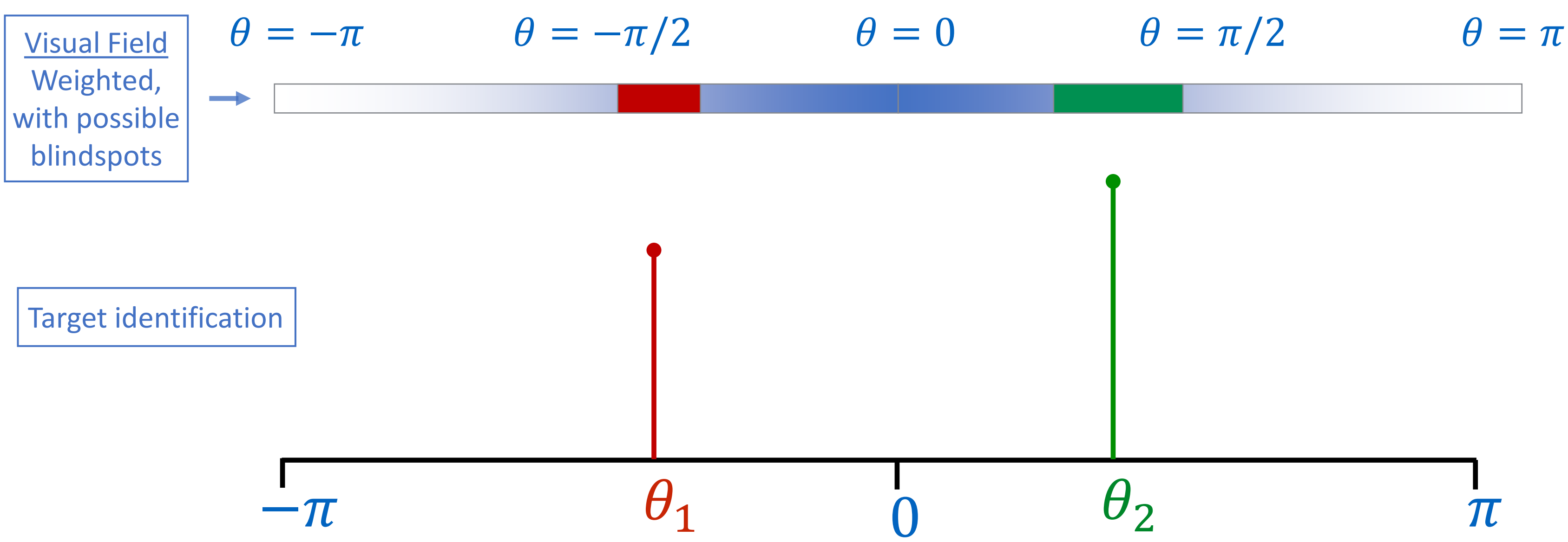
(i) Targets in a (weighted) visual field excite neurons in neural band.
(ii) Larger appearing targets excite a larger group of neurons G_k.



Organisms often have more photoreceptors in the front compared to the sides: <u>foveal vision</u>

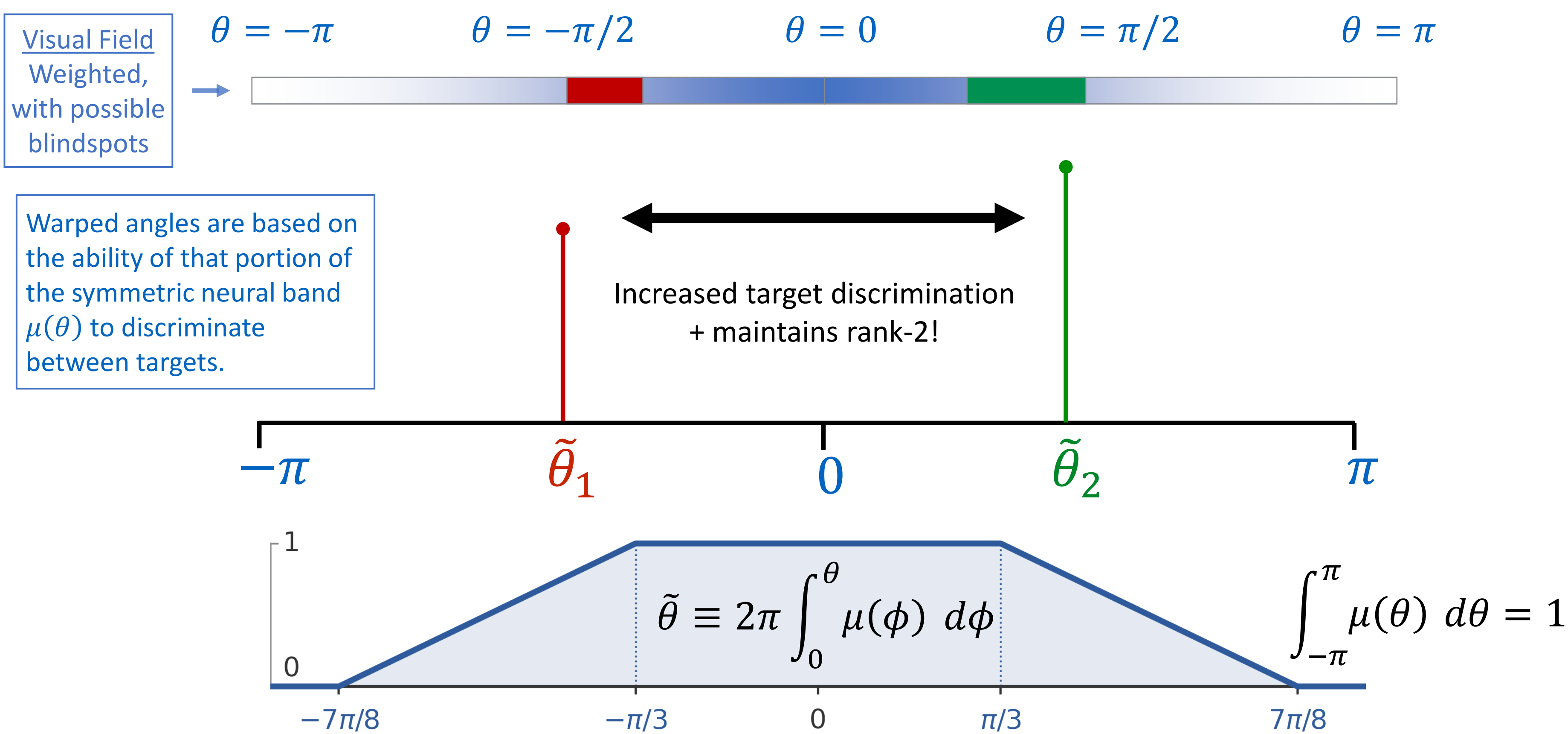
Preprocessing: Target Identification

(i) Targets in a (weighted) visual field excite neurons in neural band.
(ii) Larger appearing targets excite a larger group of neurons G_k.
(iii) Targets are identified by their center point (egocentric angle).



Preprocessing: Neural Warping

(i) Targets in a (weighted) visual field excite neurons in neural band.
(ii) Larger appearing targets excite a larger group of neurons G_k.
(iii) Targets are identified by their center point (egocentric angle).
(iv) Locations are warped to neural angles based on neural density.



Turn the modeling crank with a cosine kernel…

Let $\rho_k = |G_k|/N$ be the fraction of the total number of neurons associated with the k^{th} target.

Let $\beta = \dfrac{\mathcal{E}}{k_B T}$ be the normalized noise parameter and let $\tau = \dfrac{1}{\tau_0}$.

Then:

$$\frac{d\gamma}{d\tau} = \sum_{k=1}^{K} \frac{\rho_k e^{i\tilde{\theta}_k}}{1 + \exp\left(-2\beta \mathrm{Re}[\gamma e^{-i\tilde{\theta}_k}]\right)} - \gamma$$

where

$$\gamma = Re^{i\Theta} \equiv \sum_{k=1}^{K} n_k e^{i\tilde{\theta}_k}, \quad n_k = \frac{1}{N} \sum_{\sigma_l \in G_k} \sigma_l$$

This is a gradient flow, extending Sridhar *et al.* to include weighted targets (ρ_k) and warped neural angles ($\tilde{\theta}_k$).

Letting $\tau \to \infty$, we arrive at an equilibrium solution γ^*. We call Θ^* the *consensus angle* and R^* the *coherence strength*.

Postprocessing: Navigation

(i) In egocentric angular coordinates, we assume the observer updates its heading (on a slower timescale) with rate of change

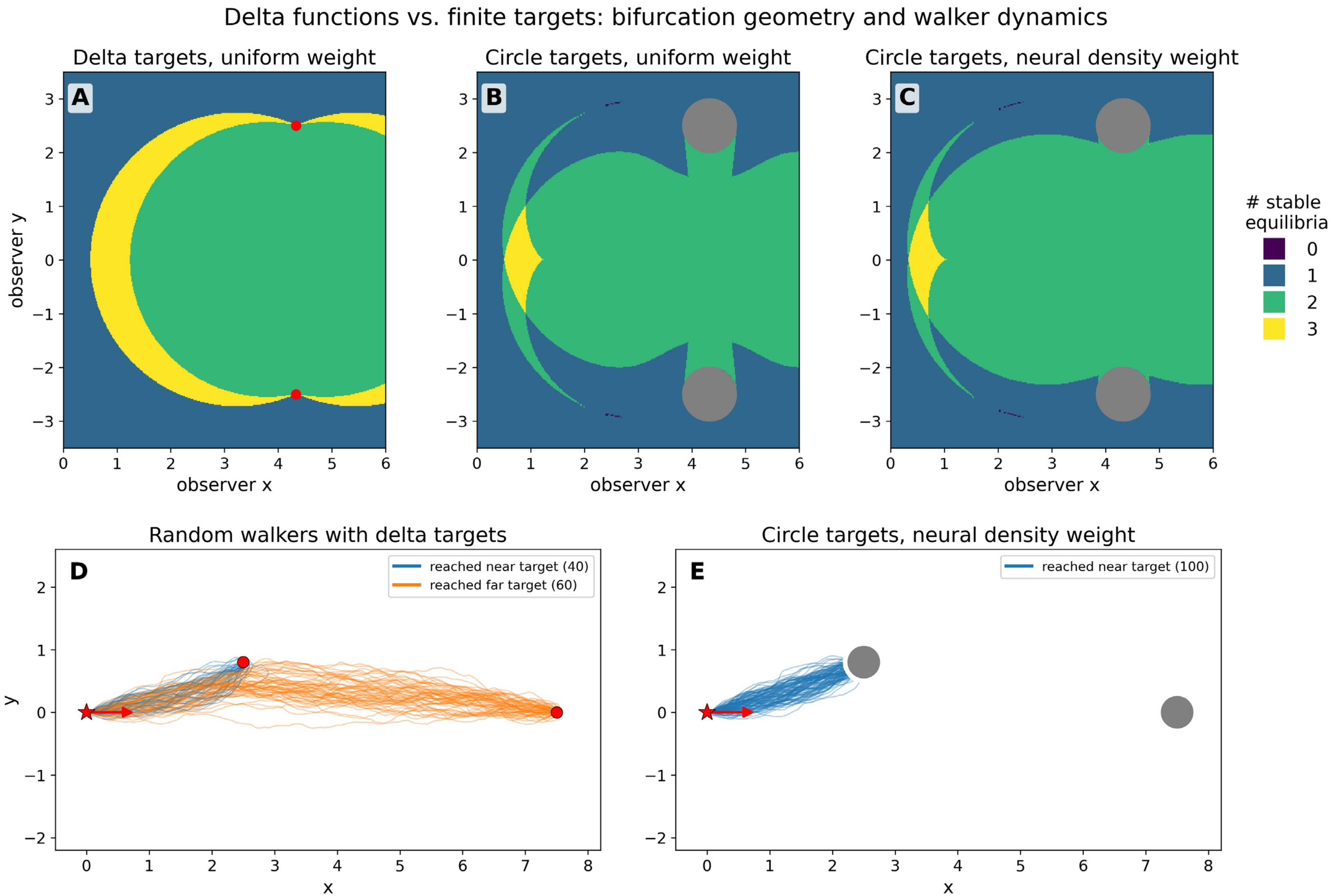
$$\frac{d\theta}{dt} = \kappa R^* \sin(\Theta^*/2), \quad \kappa \in \mathbb{R}$$

Since updates in observer heading change visual target locations, there is a coupling between this eqn. and $d\gamma/d\tau$. But angular heading is considered constant on the τ timescale.

(ii) Directional decision-making can be analyzed by looking at asymptotically stable, *self-consistent* equilibria. These are real, stable equilibria of γ, so that $d\theta/dt = 0$ and the equilibrium is stable in θ as well.

This allows us to numerically compute a bifurcation diagram of potential decisions in space.

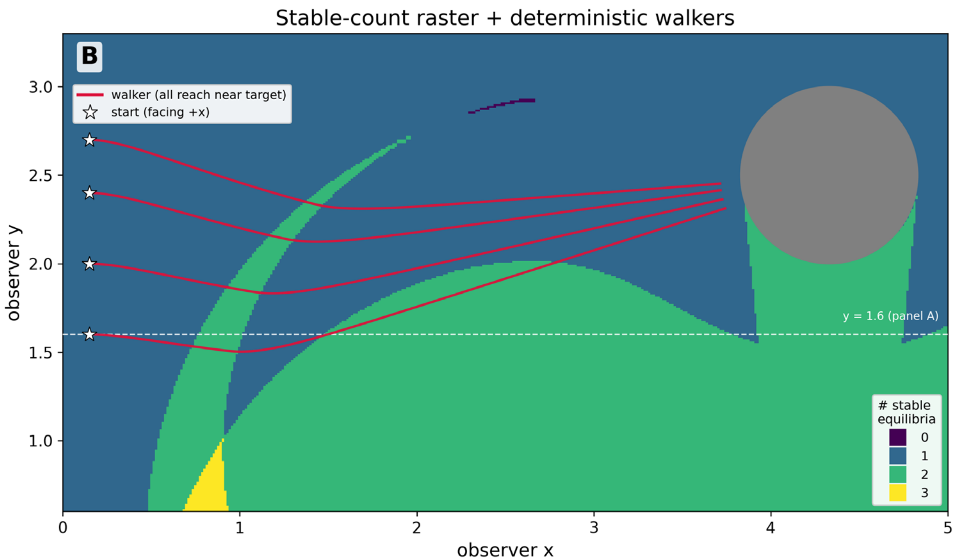
δ-targets vs. Circular Targets & Unweighted vs. Weighted

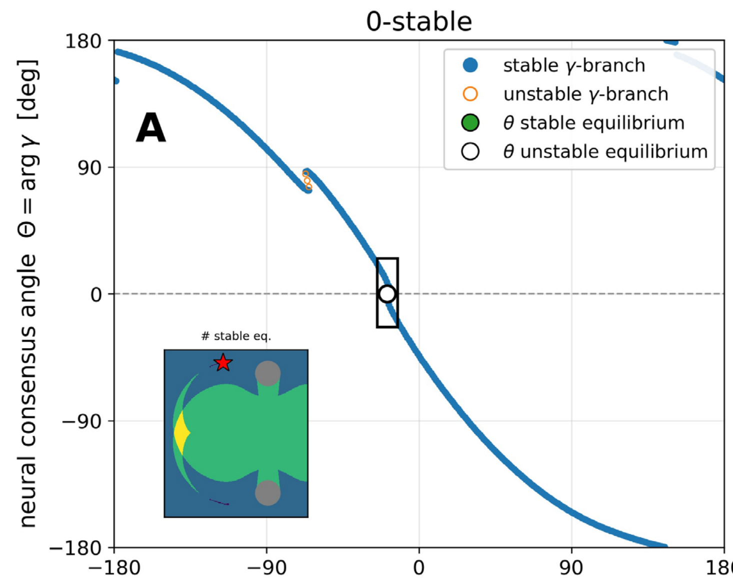


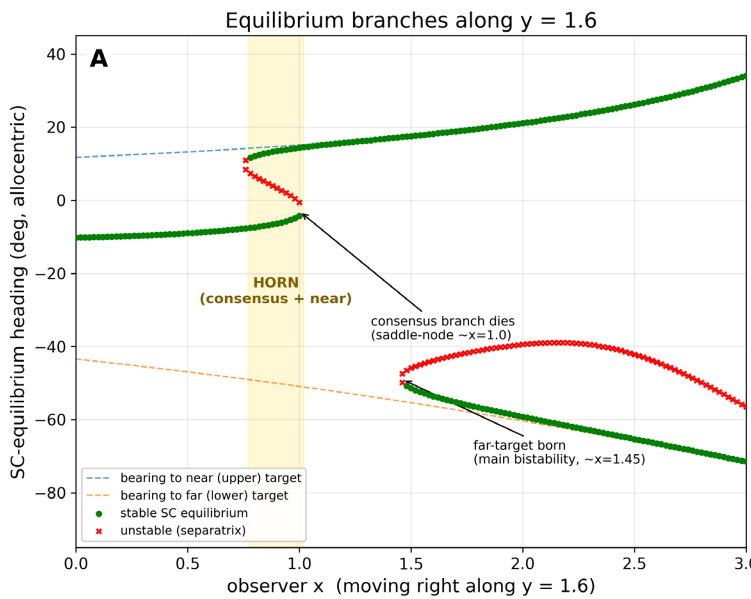
In (C), the neural density $\mu(\phi)$ was also used as the visual weight. Random walkers had additive, constant noise.

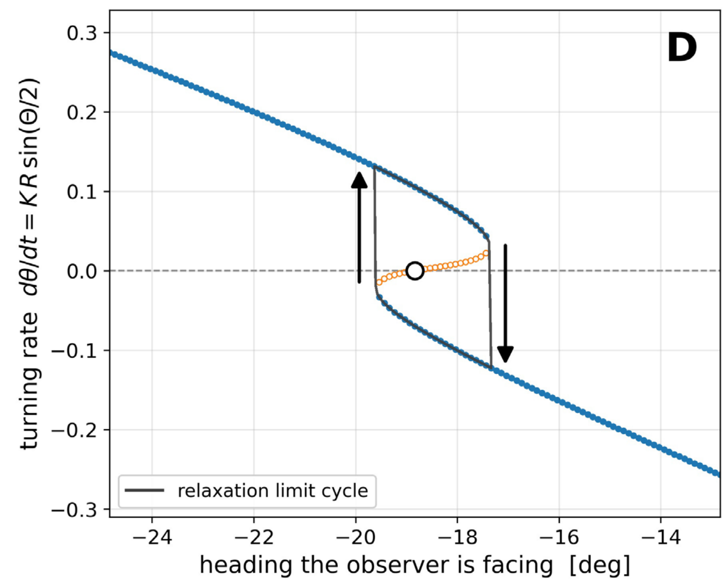
There are rich γ-bifurcations: both in space and heading!

How a deterministic walker traverses the horn and commits to the near target



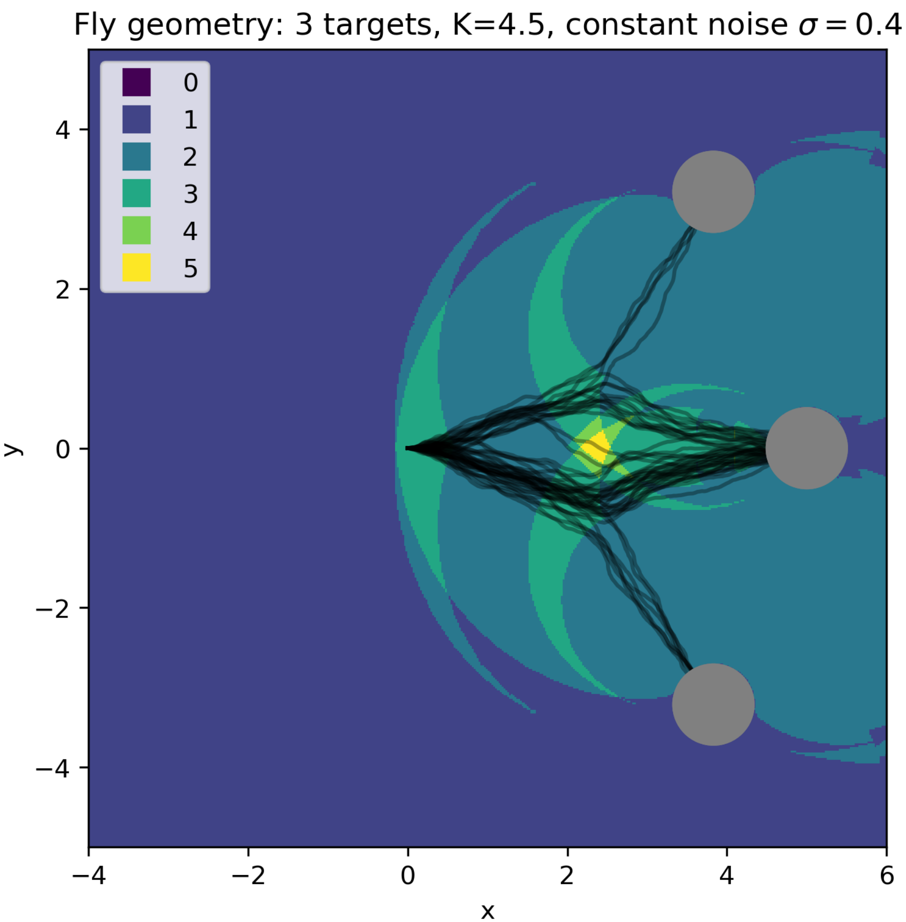


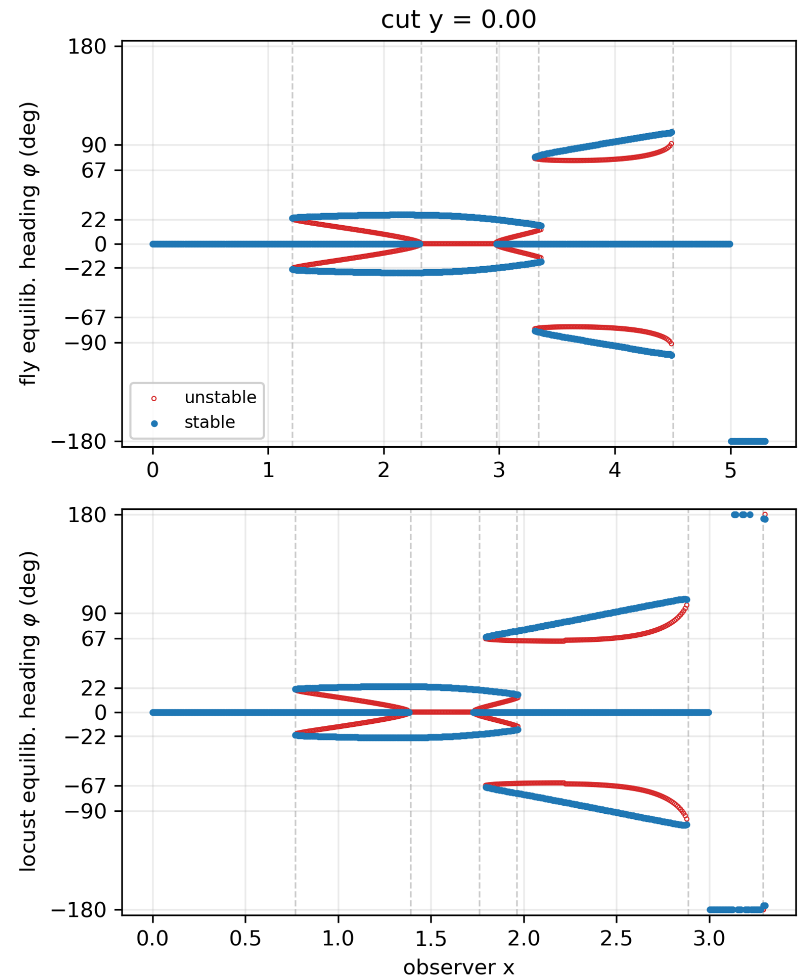




The neural density $\mu(\phi)$ controls where the bifurcation points in space occur, replacing the need to modify the interaction kernel.

But 3-target geometries **are not** a sequence of choice elimination in our framework. The center target tends to attract walkers due to the bifurcation structure.



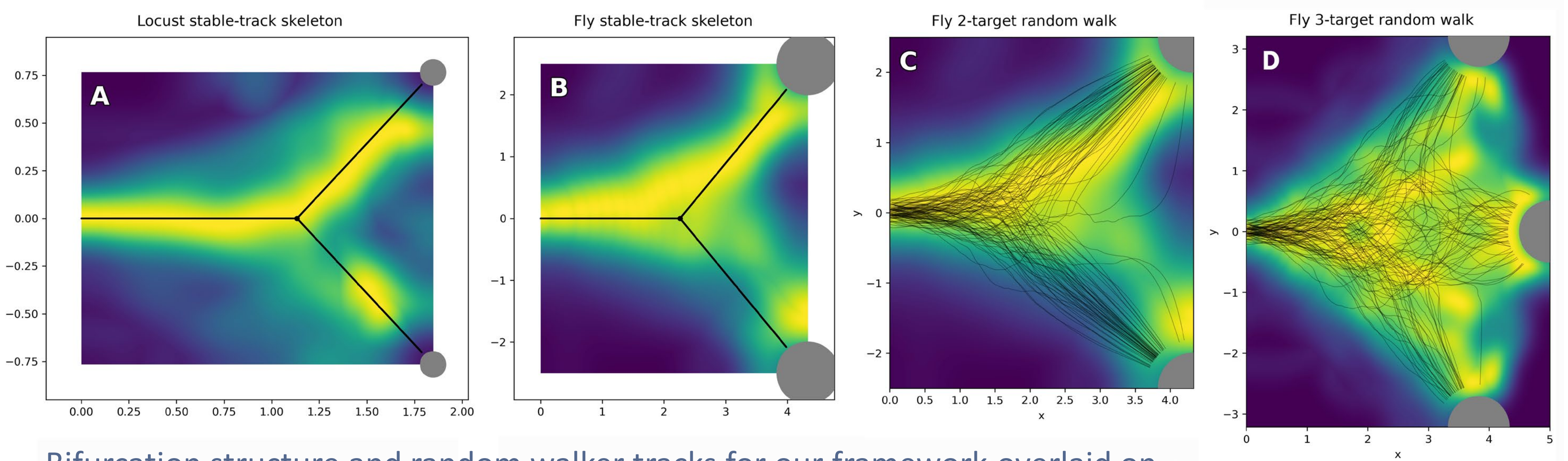


Adding noise: a Langevin model

We assume noise in γ averages out. But in θ, we want to balance target seeking behavior with searching behavior:

$$d\theta = \kappa(R^*)^p \sin(\Theta^*/2)\ dt + \sigma(1 - R^*)^q \cos(\Theta^*/2)\ dW$$

With this model, we can largely reproduce experimental data for 3-target flies, but not 3-target locusts. 2-targets are no problem!



Bifurcation structure and random walker tracks for our framework overlaid on experimental data histograms from Sridhar et al (2021).

Note: Locusts exhibit stop-and-go motion as well as jumps/hops. Strict separation of timescales and a continuous model of motion may be the wrong approaches vs. flies!

Thanks for listening!

We plan to have a preprint on arXiv soon:

Strickland, WC & Bernoff, AJ. *A weighted model of perception and decision-making between unequal targets of finite size* (working title).

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